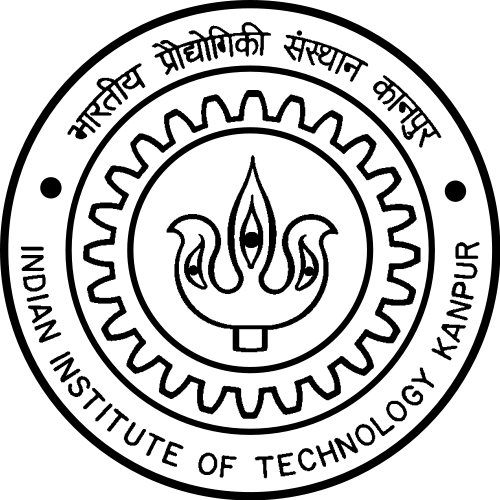
**ME685A**

**Applied Numerical Methods**

**Assignment - 4**

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**DEPARTMENT OF MECHANICAL ENGINEERING**

**IIT KANPUR**

**Question 1**

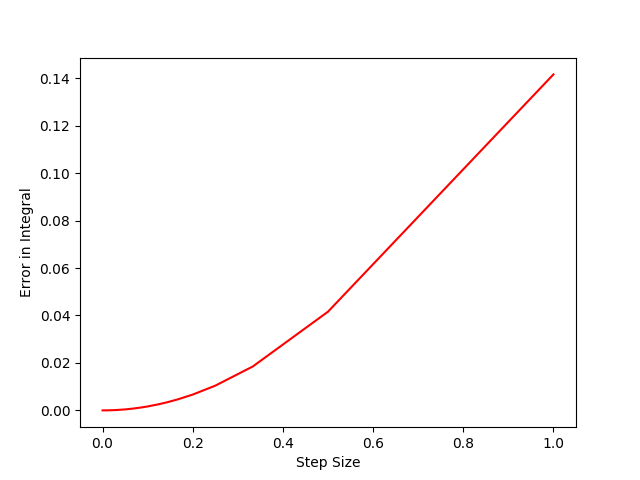
Trapezoidal rule :- Approximating function f(x) with linear interpolation

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First we conduct trapezoidal integration taking N = 1.

Then we increase the value of N upto 10000, one count at a time.

The error in integral vs step size is plotted below:

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Romberg Integration:-

We improve the accuracy of trapezoidal integration by Romberg's method

J11 = Trapezoidal integration with step size h

J11 = Trapezoidal integration with step size h/2

J11 = Trapezoidal integration with step size h/4

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J33 is the answer

Implementing Romberg integration, we get the value as 3.14159265359

From trapezoidal method we get value as 3.14159265357

The analytic integration is π = 3.14159265359

The step size taken was h = 1e-5

We find that Romberg integral is more effective than trapezoidal method

Student's data: -

Here we take the integral

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The Actual value is 1.71828182846. h used is 1e-5

Results: -

1. By trapezoidal Integration: - 1.71828182847
2. Using Romber integration: - 1.71828182846

**Question 2**

Here we use two point gauss-quadrature for evaluation of the integral

Given integral:

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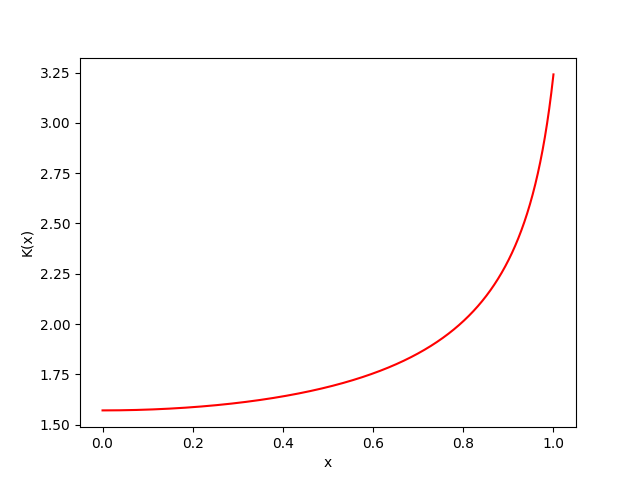
We modify the values accordingly to get it in the range (-1,1)

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By two point gauss quadrature, we get the weight functions and the Integration points as

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We plot the integral K(x) as a function of x

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**Question 3**

Given Integral: -

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Bringing them in limits -1 to 1, we get

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We use simpons formula for integration to get:

I = 1.00349034443

For Integration over a circle

We switch to polar coordinates and we obtain the integrand to be

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Between the limits of -1 to 1 for both r and , we get

I = 0.675708489837

**Question 4**

We have

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We take h = 0.1 in all cases

Euler’s Method

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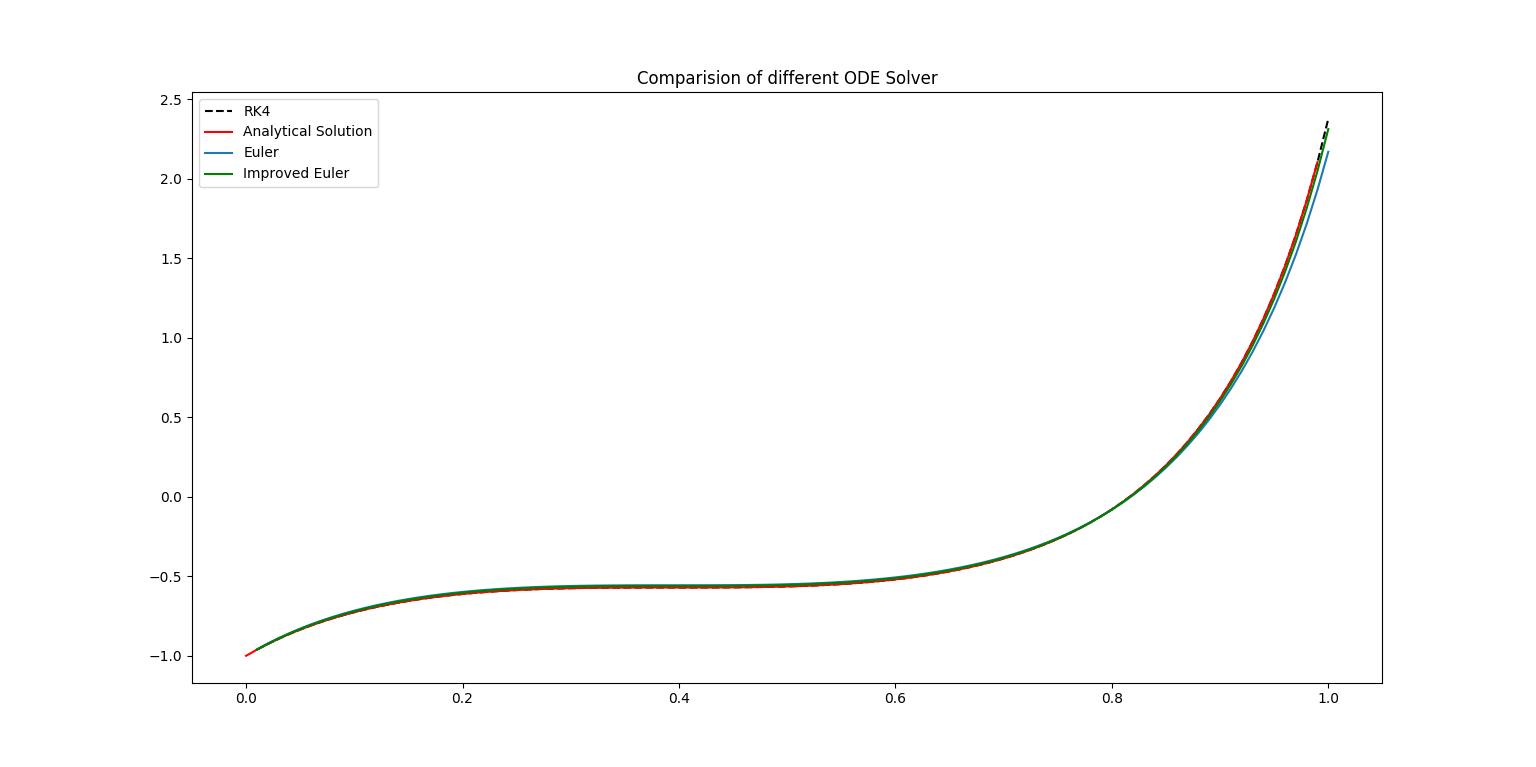
Improved Euler’s Method

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**RK4 method: -**

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We solve the ODE using above three methods and plot the graph

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**Question 5**

Lotka-Volterra predator-prey system(y1-> Rabbits y2-> Foxes)

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For no change:

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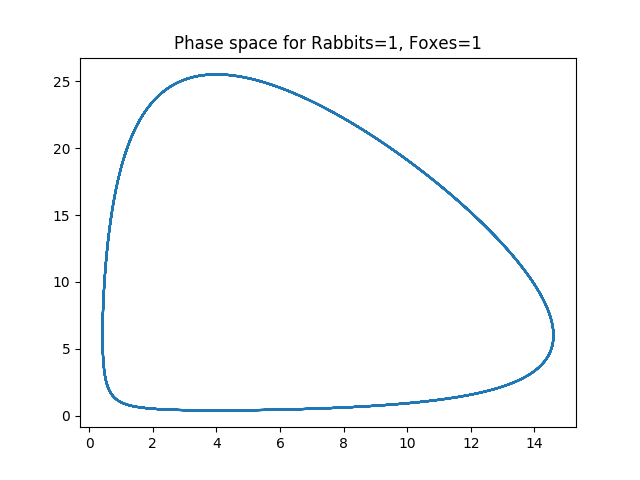
which gives

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We take a=6, b=1, c=2, d=8 and solve the equations using RK4 method for various values of initial conditions

The following graph is obtained for y1(0)=1, y2(0)=100

The phase space obtained for y1(0)= y2(0) = 1.



**Question 6**

We have

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We obtain the coefficient matrix as

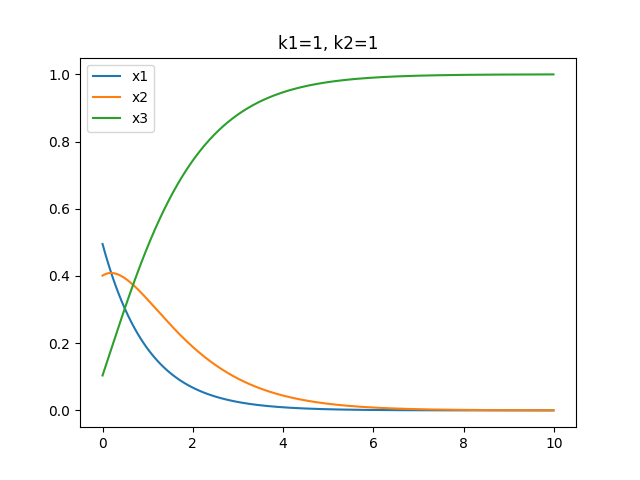
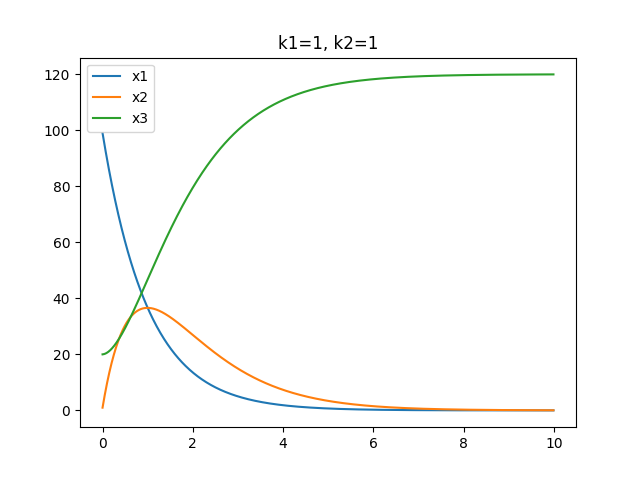
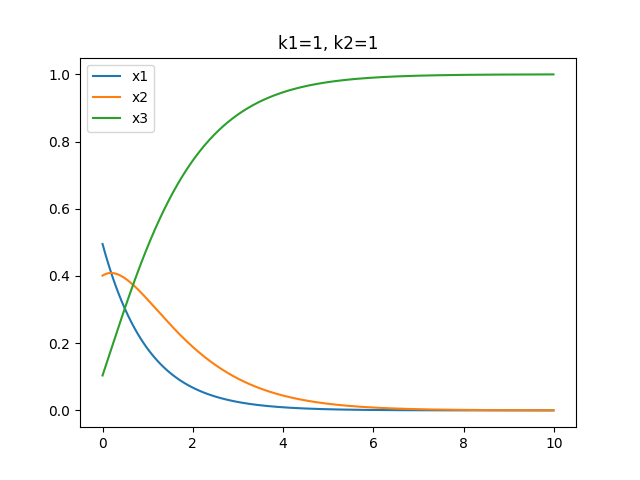
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The system becomes stiff as condition number of A increases

For stiff systems, we use Semi Implicit Euler’s Approach

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Plots:

1. X1 = 1/2 ; X2 = 2/5 ;X3 = 1/10 
2. X1 =100 ; X2 = 0 ; X3 = 0(student data)
3. Stiff System; x1 = 100; x2 = 0; x3 =0

**Question 7**

The Boundary Value Problem

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|  |  | (1) |
|  |  | (2) |

**By shooting Method: -**

We take y'(0) = p and find the error in y(1), so now error is a function of p. So we newton's method to make error zero giving p = 0.472464852127

We now plot y vs x 